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The transport properties of magnetic alloys with multi-channel Kondo impurities

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Abstract. In the framework of conformal field theory I calculate the dynamical magnetic susceptibility and electronic self-energy component at low temperatures and energies $\ll T_K$ (the Kondo temperature) for the N-channel magnetic impurities with $S < N/2$ (S is an impurity spin). The scaling dimensions of electrical resistivity, Hall resistivity and decreasing of superconducting temperature are also calculated.

1. Introduction

In 1980 Nozières and Blandin predicted the existence of the Kondo effect with a scaling low-temperature behaviour of physical properties. Such an effect should occur when the number of scattering channels, N, of the conduction electrons exceeds $2S$ (S is the impurity spin). The exact solution of the corresponding model, called the multi-channel Kondo model, was obtained some years later (Wiegmann and Tsvelick 1984, Tsvelick 1985).

The Hamiltonian of the multi-channel model is

$$H = \sum_{k,m,\sigma} \varepsilon_k C^*_{km\sigma} C_{km\sigma} + (J/N_0) \sum_{p,k,m,\sigma,\sigma'} C^*_{km\sigma} \sigma^a_{\sigma\sigma'} C_{pm\sigma'} S^a \tag{1}$$

where $m = 1, \ldots, N$, $\sigma = \pm \frac{1}{2}$, $\boldsymbol{\sigma}^a$ ($a = 1, 2, 3$) are the Pauli matrices, $C^*_{km\sigma}$, $C_{km\sigma}$ are the creation and annihilation operators of the conduction electrons, S^a are the impurity spin operators. I consider the case where $N > 2S$.

Now there is clear experimental evidence of a low-temperature scaling for magnetic impurities of vanadium in gold (Geens *et al* 1987).

A vanadium impurity ion in a strong cubic crystal field has a zero total angular moment and spin $S = \frac{3}{2}$. From comparison of the experimental data with theoretical predictions, Geens and co-workers have deduced that the number of scattering channels in AuV alloy is $N = 5$.

There are some other candidates for the non-Fermi-liquid low-temperature behaviour: U impurities in (U, Th) Be_{13} alloy; Tb impurities in **Tb**Th alloy. For these systems the theory predicts $N = 2$, $S = \frac{1}{2}$ (Cox 1987, 1988). In addition to the scaling behaviour, these alloys are likely to exhibit a new type of Kondo effect (Cox 1987, 1988). The point is that in these alloys where $J = 4$, by Hund's rule, a ground-state multiplet of an impurity ion may be split in the cubic crystal field so that the non-magnetic Γ_3 doublet lies the

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lowest. This level has a net electric quadrupole moment but no magnetic dipole moment. The Γ_3 doublet is described by the wavefunctions

$$\Psi_{1/2} = 0.54(|4\rangle + |-4\rangle) - 0.65\,|0\rangle$$

$$\Psi_{-1/2} = 0.71(|2\rangle + |-2\rangle)$$

where $|j\rangle \equiv |J_j\rangle$ is an eigenfunction of J^2 and J^z operators with $J = 4$ and its z projection j.

The Γ_3 doublet interacts with the following partial waves of the conduction electrons:

$$\left.\begin{aligned} |\Gamma_8, 2\rangle &= 0.91\,|\tfrac{5}{2}\rangle + 0.41\,|-\tfrac{3}{2}\rangle \\ |\Gamma_8, 1\rangle &= |\tfrac{1}{2}\rangle \end{aligned}\right\} [8]$$

$$\left.\begin{aligned} |\Gamma_8, -2\rangle &= 0.91\,|-\tfrac{5}{2}\rangle + 0.41\,|\tfrac{3}{2}\rangle \\ |\Gamma_8, -1\rangle &= |-\tfrac{1}{2}\rangle \end{aligned}\right\} [\bar{8}].$$

The scattering processes change only the states belonging to the same group ([8] or [$\bar{8}$]), so these states form a representation of the pseudo-spin $\frac{1}{2}$.

The most remarkable property of the quadrupole Kondo effect is that the Hamiltonian (1) in this case is a time reversal. Now the conserved index m corresponds to real spin and $\boldsymbol{\sigma}^a$, and so the S^a become pseudo-spin operators that act in the space of crystal-field eigenstates with the same spin projection. Therefore, for example, the application of an unaxial strain would produce the same effect as the Zeeman term for the normal Kondo impurity.

Another physically realisable example of the application of the model (1) has been proposed by Vladar and Zawadowski (1983) and Muramatsu *et al* (1986). According to these authors, the model (1), with $N = 2, S = \frac{1}{2}$, describes the scattering of the conduction electrons on the two-level systems in metallic glasses.

As has been mentioned above, in the model (1), low-temperature scaling takes place. This means that the renormalisation group trajectory of this model has a stable fixed point at finite coupling constant. Any theory for such a fixed point possesses the property of conformal invariance. But it is well known that impurity models are effectively one-dimensional and in this case the conformal symmetry is extreme. Recently it has been shown (Belavin *et al* 1984) that the conformal invariance puts such strong restrictions on the Green functions of one-dimensional theories that it becomes possible to calculate them.

In the present paper I use the conformal field theory to calculate the dynamical and transport properties of the multi-channel Kondo model (1).

2. The operator content of the multi-channel Kondo model

In this section I briefly discuss some general properties of conformal theories.

Until now all results have been obtained for homogeneous models. Therefore, I shall begin the discussion with such models. Below I shall explain why one can use the conformal theory for the inhomogeneous model (1).

The simplest definition of the 1 + 1-dimensional conformal theory may be given as follows. It is a theory with linear spectrum $\varepsilon = \pm vp$ whose operators are expressed in terms of free bosonic fields. This representation may be simple—then one can use it to

calculate the Green functions—or it may be very complicated—then it is better to resort to a more formal procedure, elaborated in the framework of conformal group theory.

The elementary excitations in conformal theories are waves travelling to the left and to the right. The waves travelling to the left do not interact with those travelling to the right; therefore the correlation functions depend on the coordinates $z = vt - x$ and $\bar{z} = vt + x$ taken separately. Thus, the Green functions realise the representation of the two conformal groups: the group of analytical (z) and the group of anti-analytical ($\bar{z}$) transformations. In principle one may ignore the fact that z and $\bar{z}$ are complex conjugates and consider them as independent variables. This property of general conformal theory makes it applicable to models with one impurity. In this case the impurity Green functions depend on time, t, only, and the conformal group is the group of analytical transformations of t.

In order to calculate the Green functions, one needs to know a representation of the conformal theory which they realise. The first step in finding this representation is finding the so-called 'central charge', C, of the theory. This quantity determines the universality class of the theory.

The central charge has a rather transparent physical meaning: it represents an effective number of degrees of freedom of the theory. It follows from the formula derived by Cardy (1986) which relates the heat capacity of 1+1-dimensional conformal theory to its central charge C:

$$C_v/L = \tfrac{1}{6}\pi TC \tag{2}$$

(L is a length of the system; I used $v = 1$).

When C is an integer it is clear that there are C kinds of boson in our theory. In order to obtain a theory with non-integral C one should put some restrictions on the theory with integral C. These restrictions must agree with conformal invariance. They lead to some eigenstates of the previous theory with integral C being forbidden.

The value of central charge for our model may be extracted from my previous paper (Tsvelick 1985). It is equal to

$$C = 3N/(N+2). \tag{3}$$

Here I should give some explanation. It is obvious that in the absence of the impurity the model (1) is also conformal and its central charge is $C = 2N$. Why is it not equal to this in the presence of the impurity? The point is that there are a definite proportion of the degrees of freedom that do not interact with the impurity. This fact is clearly revealed by the Bethe *ansatz* solution (the decoupled parts of the excitations are called 'charge' excitations in the earlier paper (Tsvelick 1985)). To calculate the central charge one should calculate the heat capacity of those parts of the host which couple with the impurity.

In the earlier paper (Tsvelick 1985) I found another quantity—the impurity heat capacity at $N = 2S$ (see formulae (8), (49) in this paper). But the heat capacity of the host C^{host} may be easily restored through the Fermi liquid relation

$$C^{\text{host}}/L = C^{\text{imp}}\, T_K/2$$

(T_K is the Kondo temperature), which holds in this case. As a result one obtains C^{host} given by the formula (2) with C given by (3).

The central charge (3) is just the central charge of a model well known in field theory, the Wess–Zumino–Novikov–Witten (WZNW) model on the SU(2) group. The WZNW

model has been solved in the framework of the conformal theory by Knizhnik and Zamolodchikov (1984). The model (1) belongs to its universality class.

In order to calculate the correlation functions of some physical operators of interest to us, we need to know the position of these operators in the general operator content of the theory. Therefore, I shall describe the operator content of the WZNW model.

Firstly, there are a number of so-called primary fields that transform in the simplest way under conformal group transformations. The calculation of their scaling dimensions is the main task of the theory. For the WZNW model on the SU(2) group it is known that these fields $\varphi^{(j)}(t)$ realise the $2j$th symmetrical representation of the SU(2) group and their scaling dimensions are (Knizhnik and Zamolodchikov 1984)

$$\Delta^{(j)} = j(j+1)/(N+2) \qquad j = \tfrac{1}{2}, 1, \tfrac{3}{2}, \ldots, \tfrac{1}{2}N. \tag{4}$$

Secondly, there is a set of operators ('descendants') for each primary field. They have scaling dimensions

$$\Delta^{(n,j)} = \Delta^{(n)} + n \qquad (n \text{ is an integer}). \tag{5}$$

The state with a given n is strongly degenerate.

Let us try to identify some operators of our model with enumerated conformal fields.

From the thermodynamics of the multi-channel Kondo model we know the scaling dimension of the magnetic field: $\Delta_H = N/(N+2)$ (Tsvelick 1985). Therefore, the dimension of the spin operator is $\Delta_S = 1 - \Delta_H = 2/(N+2)$. It corresponds to the primary field $\varphi^{(1)}$. It is rather obvious that spin cannot be a pure primary field. For example, from the formula for the magnetic susceptibility (Tsvelick 1985)

$$T\chi(T) = (T/T_K)^{4/(N+2)} \sum_{m=0}^{\infty} a_m (T/T_K)^{2m} \tag{6}$$

we see that the spin field consists of the $\varphi^{(1)}$ primary field and its descendants with scaling dimensions $\Delta_{(1,m)} = \Delta^{(1)} + m$. But I shall neglect the contribution of the descendants in the leading order in $(T/T_K) \ll 1$.

The spin operators may be represented as

$$S^a = d^*_\alpha \sigma^a_{\alpha\beta} d_\beta \qquad \sum_{\alpha=1}^{2} d^*_\alpha d_\alpha = 1 \tag{7}$$

where d^*_α, d_α are Fermi operators. Therefore, it is reasonable to suggest that the fermionic impurity field is transformed according to the fundamental representation of the SU(2) group. Thus, its scaling dimension is equal to

$$\Delta^{(1/2)} = \tfrac{3}{4}(N+2). \tag{8}$$

It is convenient to use for Green functions of the WZNW model on an SU(2) group the representation introduced by Zamolodchikov and Fateev (1986):

$$\varphi^{(j)}(y,t) = \sum_{m=-j}^{j} \Phi^{(j)}_m(t)[C^{m+j}_{2j}]^{-1/2} y^{(m+j)} \qquad C^k_n = n!/k!(n-k)!. \tag{9}$$

Below I use the two- and three-point correlation functions; they are calculated in the same way:

$$\langle \Phi^{(j_1)}(y_1,t_1)\Phi^{(j_2)}(y_2,t_2)\rangle = \delta_{j_1,j_2}(y_{12})^{2j_1}(t_{12})^{-2\Delta^{(j_1)}} \tag{10a}$$

$$\langle \Phi^{(j_1)}(y_1, t_1)\varphi^{(j_2)}(y_2, t_2)\Phi^{(j_3)}(y_3, t_3)\rangle = C(j_1, j_2, j_3)$$
$$\times (y_{12})^{j_1+j_2-j_3}(y_{13})^{j_1+j_3-j_2}(y_{23})^{j_2+j_3-j_1}(t_{12})^{\Delta_3-\Delta_1-\Delta_2}$$
$$\times (t_{13})^{\Delta_2-\Delta_1-\Delta_3}(t_{23})^{\Delta_1-\Delta_2-\Delta_3} \qquad (10b)$$

where $C(j_1, j_2, j_3)$ is a known structure constant.

The formulae (10) give the correlation functions on the infinite plane of the complex variable t. I need the temperature Green functions; therefore I rewrite these formulae for the infinite strip of width $1/T$. The appropriate relation for the two-point correlation function is (Belavin *et al* 1984):

$$G_T(t_1, t_2) = G_0(w(t_1), w(t_2))(\mathrm{d}w(t_1)/\mathrm{d}t_1)^{\Delta}(\mathrm{d}w(t_2)/\mathrm{d}t_2)^{\Delta} \qquad (11)$$

where $w(t) = \exp(2\pi tT)$ is the conformal transformation from the strip to the plane. The generalisation of this formula for the case of multi-point correlators is obvious.

From (10*a*) and (11) it follows that

$$\langle \Phi^{(j_1)}(y_1, t_1)\Phi^{(j_2)}(y_2, t_2)\rangle_T = \delta_{j_1 j_2}(y_{12})^{2j_1}(\pi T/\sin \pi T t_{12})^{2\Delta^{(j_1)}} \qquad (12)$$

3. The calculation of the Green functions

In the leading order in T/T_K it is easy to identify the dynamical magnetic susceptibility:

$$\chi(t_1, t_2) = A[\theta(t_{12})\langle \Phi_1^{(1)}(t_1)\Phi_{-1}^{(1)}(t_2)\rangle + \theta(t_{21})\langle \Phi_{-1}^{(1)}(t_2)\Phi_1^{(1)}(t_1)\rangle] \qquad (13)$$

where A is some constant.

From formulae (9) and (10*a*) one sees that

$$\langle \Phi_{-1}^{(1)}(t_2)\Phi_1^{(1)}(t_1)\rangle = \langle \Phi_1^{(1)}(t_2)\Phi_{-1}^{(1)}(t_1)\rangle. \qquad (14)$$

Then it is easy to obtain, using (12), the Fourier transformation of the Green function (13):

$$T\chi(\omega_n) = a_0(T/T_K)^{2\Delta^{(1)}} \frac{\cos(\omega_n/2T)\Gamma^2(1-\Delta^{(1)})}{\Gamma(1-\Delta^{(1)}+\omega_n/2\pi T)\Gamma(1-\Delta^{(1)}-\omega_n/2\pi T)} \qquad (15)$$

(here I use the formula (6) to extract the numerical factor a_0).

The retarded magnetic susceptibility is an analytic function of ω:

$$T\chi_R(\omega, T) = a_0(T/T_K)^{4/(N+2)} \frac{\cosh(\omega/2T)\Gamma^2(N/N+2)}{|\Gamma(N/N+2) + \mathrm{i}\omega/2\pi T)|^2} \qquad (N \neq 2). \qquad (16)$$

If $N = 2$ we have

$$\chi_R(\omega) = (1/\pi^2 T_K)\ln(T_K/\max(\omega, T)). \qquad (17)$$

The Green function G_d of the $\varphi^{(1/2)}$ fields enters into the expression for the self-energy part of conduction electrons Σ_e:

$$\Sigma_e(\omega_n) = C_{imp}\rho^{-1}(G_d^{-1}(\omega_n) + \mathrm{i}\pi \operatorname{sgn} \omega_n)^{-1} \qquad (18)$$

(C_{imp} is the impurity concentration).

Formula (18) needs some comments. For the singular impurity the Green function of conduction electrons is

$$G(\omega; r, r') = G_0(\omega; r - r') + G_0(\omega; r)\Sigma_e(\omega)G_0(\omega; r') \qquad (19)$$

(the impurity is located at the point $R = 0$).

In the diagrammatic representation of Σ_e there are diagrams that can and cannot be cut over the free electron propagator $G(\omega_n, 0) = -i\pi\rho \operatorname{sgn} \omega_n$. If we designate by $\rho^{-1}G_d$ the part of Σ_e which cannot be cut, then we get from (19) the formula (18) (with $C_{imp} = 1$). The right-hand coefficient C_{imp} arises after averaging over impurities.

My point is that in the leading order in (T/T_K) one has

$$G_d(t) = \langle\langle d_\sigma(t) d_\sigma^*(0)\rangle\rangle = B\langle\langle \Phi_\sigma^{(1/2)}(t)\varphi_{-\sigma}^{(1/2)}(0)\rangle\rangle.$$

It is a fermionic Green function and therefore it is equal to

$$\begin{aligned} G_d(t) &= B[\theta(t)\langle\Phi_{1/2}^{(1/2)}(t)\varphi_{-1/2}^{(1/2)}(0)\rangle + \theta(-t)\langle\varphi_{-1/2}^{(1/2)}(0)\varphi_{1/2}^{(1/2)}\rangle] \\ &= B[\theta(t)\langle\varphi_{1/2}^{(1/2)}(t)\varphi_{-1/2}^{(1/2)}(0)\rangle - \theta(-t)\langle\varphi_{1/2}^{(1/2)}(0)\varphi_{-1/2}^{(1/2)}(t)\rangle] \end{aligned} \tag{20}$$

(in the last row I use formula (10*a*)).

The Fourier transformation of (20) is

$$G_d(\omega_n) = b_0(T/T_K)^{-1+2\Delta^{(1/2)}} \frac{(i \sin(\omega_n/2T))}{\Gamma(1-\Delta^{(1/2)} + \omega_n 2\pi T)\Gamma(1-\Delta^{(1/2)} - \omega_n/2\pi T)} \tag{21}$$

where $\omega_n = 2\pi T(n+1/2)$.

Following the standard procedure I obtain from (18), (21) the expression for the imaginary part of the retarded self-energy:

$$\begin{aligned} 1/\tau(\omega) &= -2 \operatorname{Im} \Sigma_e^R(\omega) = 2C_{imp}\rho^{-1}[1 - 1/(\pi b_0)^2 (T/T_K)^{(2N+1)/(N+2)} \\ &\quad \times |\Gamma(1 - \tfrac{3}{4}(N+2) + i\omega/2\pi T)|^2 \cosh^{-2}(\omega/2T)]. \end{aligned} \tag{22}$$

At last I calculate the function

$$\partial G_d(t, H)/\partial H|_{H=0} = BA^{1/2} \int_0^{1/T} d\tau \langle\langle \varphi_{1/2}^{(1/2)}(t)\varphi_{-1/2}^{(1/2)}(0)\varphi_0^{(1)}(\tau)\rangle\rangle. \tag{23}$$

Using the formulae (10*b*) I obtain, after cumbersome algebra, the following expressions:

$$\begin{aligned} \partial G_d(\omega_n, H)/\partial H &= D \int_0^\pi dt \sin(\omega_n t/\pi T) \cos t (\sin t)^{1/2 - 2\Delta^{(1/2)}} \\ &\quad \times F(1 - \Delta^{(1)}/2, 1 - \Delta^{(1)}/2, \tfrac{3}{2}, \cos^2 t)(T/T_K)^{(2\Delta^{(1/2)} + \Delta^{(1)} - 2)} \\ D &= 2^{1+2\Delta^{(1)}} \cos \pi\Delta^{(1)}/2 \, \Gamma(N/N+2) BA^{1/2} C(\tfrac{1}{2}, \tfrac{1}{2}, 1) \\ C(\tfrac{1}{2}, \tfrac{1}{2}, 1) &= \left(\frac{\Gamma(1/(N+2))\Gamma^2(N/(N+2))\Gamma(3(N+2))}{\Gamma((N+1)/(N+2))\Gamma^2(2/(N+2))\Gamma((N-1)/(N+2))}\right)^{1/2}. \end{aligned} \tag{24}$$

4. The calculation of the transport properties

In this section I apply the formulae for the Green functions obtained in the previous sections to calculate the impurity contribution to electrical resistivity, Hall resistivity and the depression of the superconducting transition temperature.

According to Fulde and Peschel (1972) the electrical conductivity is given by the formula

$$\sigma = \frac{e^2 V_F^2}{3} \int \frac{d^3k}{(2\pi)^3} (-\partial n(\varepsilon_K)/\partial \varepsilon_K) \tag{25}$$

where $n(\varepsilon) = [\exp(\varepsilon/T) + 1]^{-1}$ and $\tau(\varepsilon)$ is given by formula (22).

From (22) and (25) I obtain

$$\begin{aligned} \sigma(T) &= \sigma(0)(1 + \beta(T/T_K)^{(2N+1)/(N+2)} + \dots \\ \beta &= (2/\pi b_0)^2 \int_{-\infty}^{+\infty} dx \cosh^{-4} x \, |\Gamma(1 - \tfrac{3}{4}(N+2) + ix/\pi)|^2 \end{aligned} \tag{26}$$

where $\sigma(0)$ is the conductivity in the unitarity limit.

Levy *et al* (1988) derived the following formula for the Hall resistivity:

$$\begin{aligned} R_H = \rho_{xy}/H = &- \frac{36 C_{imp} g^2 \sin \eta^2}{49 e^2 \hbar H} \\ &\times \int d\varepsilon \left[\left(-\frac{\partial n(\varepsilon)}{\partial \varepsilon} \right) \rho_0(\varepsilon) V_F^2(\varepsilon) \operatorname{Im} \left(e^{-i\eta_2} \sum_\sigma \sigma \Sigma^R_{\sigma,e}(\varepsilon) \right) \rho_e^{-2}(\varepsilon) \right] \\ &\times \left[\int \left(-\frac{\partial n(\varepsilon)}{\partial \varepsilon} \right) \rho_0(\varepsilon) V_F^2(\varepsilon) \rho_e^{-1}(\varepsilon) \right]^{-2} \end{aligned} \tag{27}$$

$$\rho_e(\varepsilon) = -\operatorname{Im} \sum_\sigma \Sigma^R_\sigma(\varepsilon).$$

Here g is a matrix element of the hybridisation, $\rho_0(\varepsilon)$ is a density of states of the band electrons, $V_F(\varepsilon)$ is the Fermi velocity, η_2 is a scattering phase of the potential scattering.

From (22), (24) and (27) I obtain at $H \to 0$

$$R_H \sim C_{imp} \frac{\sin 2\eta_2}{e m_e (\rho V_F)^2 T_K} (T/T_K)^{(2N-1)/(N+2)}. \tag{28}$$

The Hall coefficient per impurity is enormous compared to its value in the host:

$$R_H / C_{imp} R_H^{host} \sim E_F / T_K.$$

It is also interesting to consider the influence of the magnetic impurities on a superconducting transition. Using the well known formula for the reduction of the critical temperature T_c, I obtain:

$$\begin{aligned} \partial \ln T_c / \partial C_{imp} &= -\pi^2 \rho T^2 \sum_{n,m} \frac{1}{|\omega_n| |\omega_m|} \Gamma(\omega_n, \omega_m) \\ &= -\rho \frac{t}{8\pi} \int_0^{1/T} \prod_{l=1}^{4} dt_e \, \mathcal{L}(T t_{13}) \mathcal{L}(T t_{24}) \Gamma(t_1, t_2, t_3, t_4). \end{aligned} \tag{29}$$

$$\Gamma_{1234} = \langle\langle d_\uparrow(t_1) d^*_\uparrow(t_2) d_\downarrow(t_3) d^*_\downarrow(t_4) \rangle\rangle$$

$$\mathcal{L}(x) = \ln(\coth^2 \pi x/2).$$

I am not writing down the explicit expression for Γ because it is very complicated and the integral (29) cannot be calculated analytically. Instead, I calculate the scaling

dimension of $\partial \ln T_c/\partial C_{imp}$. From the paper by Zamolodchikov and Fateev (1986) it is known that

$$\Gamma_{1234} = (T/T_K)^{[4\Delta^{(1/2)}+2\Delta^{(1)}]} f(Tt_{12}, Tt_{13}, Tt_{14}). \tag{30}$$

Substituting (30) into (29) I obtain the estimate

$$\partial \ln T_c/\partial C_{imp} \sim -(T_c/T_K)^{[-3+4\Delta^{(1/2)}+2\Delta^{(1)}]} = -(T_K/T_c)^{(3N-1)/(N+2)}. \tag{31}$$

5. Conclusion

Here I briefly discuss the results. I calculate the dynamical susceptibility (16), (17), the scattering rate (22), the electrical conductivity σ (26), the Hall resistivity R_H (28) and the depression of the temperature of superconducting transition (3). The corresponding scaling dimensions are

$$\Delta_\sigma = (2N+1)/(N+2) \qquad \Delta_{R_H} = (2N-1)/(N+2) \qquad \Delta_{T_c} = (3N-1)/(N+2).$$

To calculate them I need only scaling dimensions of the two operators—$\Delta^{(1/2)}$ and $\Delta^{(1)}$. All the complicated apparatus of the conformal field theory has been required for another task—to obtain the susceptibility and the scattering rate as functions of (ω/T) in the crossover region. Therefore, the experimental observation of this dependence seems to me a very important test for the applicability of the conformal field theory to models with magnetic impurities.

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